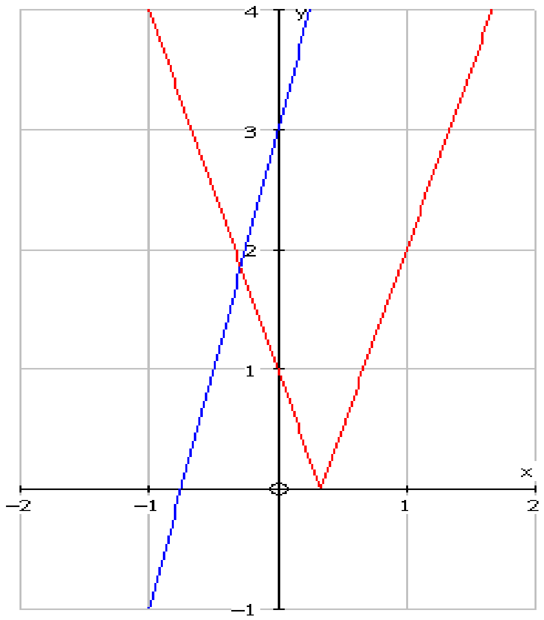
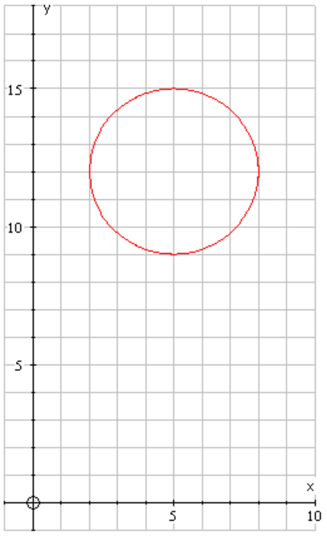
Further Pure Mathematics FP2 (6668)

Mock paper mark scheme

Question number	Scheme	Marks
1. (a)		
	Line correct	B1
	V shape correct	B1
	$\frac{1}{3}$ and $-\frac{3}{4}$	B1 (3)
(b)	Point of intersection when $4x + 3 = 1 - 3x$, and so $x = -\frac{2}{7}$	M1 A1
	Solution is $x > -\frac{2}{7}$	A1 (3)
		(6 marks)
2. (a)	$\frac{1}{2r+1} - \frac{1}{2r+3}$	M1 A1 (2)
(b)	$\sum = \frac{1}{3} - \frac{1}{5} + \frac{1}{5} - \frac{1}{7} + \ldots \quad + \frac{1}{2n+1} - \frac{1}{2n+3}$	M1 A1
	$= \frac{1}{3} - \frac{1}{2n+3} = \frac{2n+3-3}{3(2n+3)} = \frac{2n}{3(2n+3)}$ (*)	A1 cso (3)
		(5 marks)

Question number	Scheme	Marks
3. (a)	$\frac{dy}{dx}=\frac{5}{1+5x}$, $\frac{d^2y}{dx^2}=-\frac{25}{(1+5x)^2}$, $\frac{d^3y}{dx^3}=\frac{250}{(1+5x)^3}$	M1 A1, A1 A1 (4)
(b)	$\ln(1+5x)=5x-\frac{25}{2}x^2+\frac{125}{3}x^3+...$	M1 A1 A1 (3)
		(7 marks)
4.	$\frac{d^2y}{dx^2}+1+1=4$ at $x = 0$, $\therefore \frac{d^2y}{dx^2}=2$	B1
	Differentiate to give	
	$\frac{d^3y}{dx^3}+[\left(\frac{dy}{dx}\right)^2+y\frac{d^2y}{dx^2}]+2y\frac{dy}{dx}=3$	M1 [M1 A1] A1
	At $x = 0$, $\frac{d^3y}{dx^3}+[1^2+1\times 2]+2=3$ and $\frac{d^3y}{dx^3}=-2$	B1
	$y=1+x+\frac{2x^2}{2}-\frac{2x^3}{6}+...$	M1 A1
		(8 marks)
5.	Area = $\frac{1}{2}\int_0^{\frac{\pi}{2}}(4+4\sin 3\theta+\sin^2 3\theta)\ \ d\theta$	M1
	$=\frac{1}{2}\left[4\theta-\frac{4\cos 3\theta}{3}+\frac{\theta}{2}-\frac{\sin 6\theta}{12}\right]_0^{\frac{\pi}{2}}$	M1 A1 M1 A1
	$=\frac{1}{2}\left(2\pi+\frac{\pi}{4}\right)-\frac{1}{2}\left(-\frac{4}{3}\right)$	M1
	$=\frac{9\pi}{8}+\frac{2}{3}$	A1 (7)
		(7 marks)

Question number	Scheme	Marks
6. (a)	$\mathrm{i}\sin 5\theta = \mathrm{Im}(\cos\theta + \mathrm{i}\sin\theta)^5$	M1
	$= \mathrm{i}(5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta)$	M1 A1
	$= \mathrm{i}(5(1-\sin^2\theta)^2\sin\theta - 10(1-\sin^2\theta)\sin^3\theta + \sin^5\theta)$	
	$\therefore \sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$	(5)
(b)	Put $5\sin\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$	
	$\therefore 16\sin^5\theta - 20\sin^3\theta = 0$	M1
	$\therefore \sin\theta = 0$ or $\sin\theta = \pm\sqrt{\frac{5}{4}}$ (no solution as $\sin\theta > 1$)	A1 A1
	So only solutions are $\theta = n\pi$.	A1 (4)
		(9 marks)
7. (a)	Integrating factor is $\mathrm{e}^{-\int 0.1dt} = \mathrm{e}^{-0.1t}$	B1
	Use to obtain $P\mathrm{e}^{-0.1t} = \int 0.05t\mathrm{e}^{-0.1t}\mathrm{d}t$	M1
	$= \frac{-0.05t\mathrm{e}^{-0.1t}}{0.1} + \int \frac{0.05\mathrm{e}^{-0.1t}}{0.1}\mathrm{d}t$	M1
	$= -0.5t\mathrm{e}^{-0.1t} - 5\mathrm{e}^{-0.1t} + c$	A1
	$\therefore P = -\frac{1}{2}t - 5 + c\mathrm{e}^{0.1t}$	A1
	But at $t = 0$, $P = 10000$	
	So $c = 10005$ and $\therefore P = -\frac{1}{2}t - 5 + 10005\mathrm{e}^{0.1t}$	M1 A1 (7)
(b)	When $t = 6$, $P = 18222 < 20000$	
	When $t = 7$, $P = 20139 > 20000$	M1
	So P reaches 20 000 during the seventh year..	A1 (2)
		(9 marks)

Question number	Scheme	Marks
8. (a)		
	Locus is a circle	B1
	Centre is at (5, 12)	B1
	Radius is 3	B1 (3)
(b)	Finds distance from centre to origin is 13	M1
	Maximum modulus is 13 + 3 = 16	M1 A1
	Minimum modulus is 13 – 3 = 10	A1(4)
(c)	Finds $\arctan \frac{12}{5}$	M1
	Uses $\arctan\frac{12}{5} \pm \arcsin\frac{3}{13}$	M1
	Obtains 0.94 and 1.41 radians	A1 A1 (4)
		(11 marks)

Question number	Scheme	Marks
9. (a)	$V = \lambda t\sin 8t$, $\quad \dfrac{dV}{dt} = \lambda\sin 8t + 8\lambda t\cos 8t$	M1, A1
	Substitute to give $\dfrac{d^2V}{dt^2} = 16\lambda\cos 8t + 64\lambda t\sin 8t$	A1
	$16\lambda\cos 8t = \cos 8t$, and $\therefore \lambda = \frac{1}{16}$	M1, A1 (5)
(b)	Auxiliary equation is $m^2 + 64 = 0$ and so $m = \pm 8\text{i}$	B1
	Complementary function is $A\cos 8t + B\sin 8t$	M1 A1
	General solution is $A\cos 8t + B\sin 8t + \frac{1}{16}t\sin 8t$	B1 (4)
(c)	$V = 0$, when $t = 0$ implies $A = 0$	
	$8B\cos 8t + \frac{1}{16}\sin 8t + \frac{1}{2}t\cos 8t = 0$ when $t = 0$	
	So $8B = 0$ and $V = \frac{1}{16}t\sin 8t$ is particular solution.	(3)
(d)	As t becomes large the amplitude of the oscillations of V become large also. As $t \to \infty$, $\quad V \to \infty$ also.	B1 (1)
		(13 marks)